BOOK OF ABSTRACTS

Curvature Bundle Morphisms for Hypersurface Generalized Dirac Operators

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The datum of a generalized Dirac operator (in the sense of Gromov and Lawson) on a Riemannian manifold can be restricted to a hypersurface, and then suitably altered via a bundle morphism-valued one-form commuting with Clifford multiplication, to yield a generalized Dirac operator on the hypersurface. How do the associated curvature bundle morphisms, on the hypersurface and on the ambient manifold, relate? The Gauss-like relationship we are going to establish in this paper involves the shape operator of the hypersurface, a normal part of the curvature associated to the connection giving the Dirac operator on the ambient manifold, and the curvature of the one-form. Applications are then given to operators of Spin and Clifford type.

Geometric Applications of Homothetic Production Functions

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A real-valued function f is homothetic if it is of the form: $f(\mathbf{x}) = h(g(\mathbf{x}))$, where g is homogeneous of any degree $\neq 0$ and h is strictly monotone [1]. The homothetic production functions satisfying Monge-Ampère equation and constant elasticity of subtitution were classified by B.-Y. Chen [2, 3].

In this talk, we study the homothetic production functions and investigate geometric properties of the graphs of the homothetic production functions.

2000 MSC: 91B38, 15A15, 53B25.

Keywords and phrases: Homothetic production function, generalized Cobb-Douglas production function, Flat space.

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On Quasi-Sum Production Functions

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A function $f : \mathbb{R}_+^n \longrightarrow \mathbb{R}_+$ is called a quasi-sum if there exist continuous strict monotone functions $g_i : \mathbb{R}_+ \longrightarrow \mathbb{R}$ $(i = 1, ..., n)$ and there exist an interval $I \subseteq \mathbb{R}$ of positive length and a continuous strict moonotone function $h : I \longrightarrow \mathbb{R}_+$ such that for every $\mathbf{x} = (x_1, ..., x_n) \in \mathbb{R}_+^n$ we have $f(\mathbf{x}) = h(g_1(x_1) + ... + g_n(x_n))$ [1]. The quasi-sum production functions were completely classified in terms of their graphs by B.-Y. Chen [2].

In this talk, we present some geometric results regarding the graphs of the quasi-sum production functions.

2000 MSC: 91B38, 15A15, 53B25.

Keywords and phrases: Quasi-sum production function, production hypersurface, flat space, Gauss-Kronocker curvature.

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Statistical Finsler-Randers Structures for the Garner Cancer Cell Model

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The present talk describes a class of statistically-built Finslerian structures which are related to the classical Garner dynamical system which models the cancer cell population growth.

It is shown that a certain localy-Minowski anisotropic Randers structure is able to provide a Zermelo-type drift of the overall cancer cell population growth, which occurs due to significant changes within the cancerous process. The geometric background, the applicative advantages and perspective openings of the constructed geometric structure are discussed.

On A New Type of Almost Contact Metric Manifolds

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In the present paper we introduce a new class of contact manifolds. Such type manifolds are called almost contact (k,μ)-spaces with cyclic-parallel τ-curvature tensor. We investigate some curvature properties of these manifolds and we obtain that these manifolds have cyclic parallel Ricci tensor under some conditions. Furthermore, we get these manifolds are K-contact under some special cases and we obtain that M is locally isometric to the product $\mathbb{E}^{n+1} \times \mathbb{S}^n(4)$ under some algebraic conditions.

Kaluza-Klein Spaces with Almost Paracontact Structures

Cornelia-Livia BEJAN

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The notion of the almost paracontact structure was introduced by I. Sato as an analogue of the almost contact structure. We show here a relation between Kaluza-Klein spaces and the almost paracontact structures.

Harmonicity on Walker Manifolds

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Let $(W, q, \mathcal{D})$ be a 4-dimensional Walker manifold. We classify all harmonic functions locally defined on $(W, q, \mathcal{D})$. The notion of quadratic map is introduced here first from a local chart of a 4-dimensional Walker manifold to the n-dimensional semi-Euclidean space of index r, and then between local charts of two 4-dimensional Walker manifolds. We obtain some necessary and sufficient conditions under which these maps are harmonic, horizontally weakly conformal, or harmonic morphisms. We provide a few characterizations and examples for several special $(1,1)-$tensor fields on $(W, q, \mathcal{D})$. Then we prove that the three endomorphisms fields of a certain hyper-para-Hermitian structure on the canonical Walker space $(\mathbb{R}^4, q)$ are harmonic w. r. t. q if and only if the structure is hyper-para-Kaehler. On the total space of the tangent bundle of W, we characterize the harmonicity of some metrics w.r.t. the Sasaki metric, the horizontal and the vertical lift of q.

On Slant Submanifolds of Semi-Riemannian Manifolds

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Slant submanifolds of an almost Hermitian manifold were introduced by B.-Y. Chen as a natural generalization of both complex and totally real submanifolds. Later, they were also defined in the almost contact metric setting. In this talk, we will review the main facts about this theory, as well as its extension to the semi-Riemannian case, by analyzing some already known notions as well as some new ideas.

Gel'fand Topologies on Maximal Spectra of Complex Subalgebras and Some Applications in 14-th Hilbert Problem Aria

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Let k be a field (e.g. $k = \mathbb{C}$).

By a k-subalgebra A we will nominate a subalgebra of a k-algebra of finite type A'. Then passing to the associated geometric objects - the schemes, we receive a dominant morphisms of k-schemes $f : X = Spec\, A' \to X^* = Spec\, A$, where X is just an algebraic k-variety.

The first main problem in 14-th Hilbert Problem aria is to find conditions under which A becomes of finite type over k, equivalently X^* becomes an algebraic k-variety.

If $k = \mathbb{C}$ then on the maximal spectrum of A, $Spec.maxA = \{m \subset A | m$ maximal ideal $\}$, resp. on the set of all closed points X^*_{cl} of a $\mathbb{C}$-scheme X^* dominated by an algebraic $\mathbb{C}$-variety, we have introduced ([1]) in parallel with *Zariski* topology a finer topology, called *Gel'fand topology*, which control completely the finite generation of the subalgebra A, resp. the algebraicity of the $\mathbb{C}$-scheme X^* :

Theorem ([1]) - *A reduced $\mathbb{C}$-subalgebra A is finitely generated iff Gel'fand topology on $Spec.maxA$ is locally compact*

Equivalently, a reduced $\mathbb{C}$-scheme X^* dominated by an algebraic $\mathbb{C}$-variety is an algebraic variety iff Gel'fand topology on X^*_{cl} is locally compact.

In this talk we illustrate the impact if these facts in the 14-th *Hilbert* Problem aria, related to a result of *Zariski*, the question of the descent of the algebraicity by some classes of morphisms of k-schemes (in particular to a Theorem of *Goodman-Landman* ([3]), or to so-called *"Zariski and Constantinescu"* Theorem (MR1836861 (2002e:14022)).

An open question related to the descent of the algebraicity by universally quotient morphisms of k-schemes, suggested by a topological result of *Mitrofan Choban*, can be raised.

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Isometric Immersions of S-Manifolds in Real Space Forms

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In this article, we investigate sharp inequalities involving new contact Riemannian invariants and the squared mean curvature for S-manifolds in real space forms.

2000 MSC: 53C40, 53C25.
Keywords and phrases: S-manifolds, squared mean curvature, contact Riemannian invariants, inequalities.

Some Equilibrium Problems with Convexity(-type) Assumptions in the Completely Metrizable Locally Convex Spaces Setting vs the Riemannian Manifolds Setting

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The study of *equilibrium problems*, often in a non-linear geometrical framework is motivated by our daily life. In the *first part* of this paper, we will consider some fixed-point, coincidence and quasi-coincidence results with (compactness and) *convexity* hypothesis, for families of multimaps in the *completely metrizable locally convex spaces setting*. We will apply these results to solve constrained equilibrium problems. Shizuo Kakutani proved in 1941 his famous fixed-point theorem for multimaps, and since then it is known that the convexity assumption on the domains is essential. In *the second part* of the paper, we will investigate what can we do when this assumption of convexity is not fulfiled. The study of some known Nash-type equilibrium results, reveals a geometric idea, namely to embed these non-convex domains into suitable *Riemannian manifolds*. So we will regain certain *geodesic convexity* property of them.

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Optimal Inequalities Involving Casorati Curvature of Slant Submanifolds in Quaternionic Space Forms

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A topic of great interest in the submanifold theory is the study of optimal inequalities between intrinsic and extrinsic invariants of submanifolds in some ambient spaces. The purpose of this paper is focused on to establish optimal inequalities involving Casorati curvature and scalar curvature of slant submanifolds in quaternionic space forms. The equality cases are also investigated. The Casorati curvature of a submanifold is an extrinsic invariant, concept preferrated by F. Casorati over the traditional Gauss curvature and the mean curvature because it corresponds better with *the common intuition of curvature*.

Keywords and phrases: Casorati curvature, quaternionic space forms, optimal inequality, Casorati ideal submanifold.

Some Inequalities for Riemannian Submersions

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In this paper, we compute scalar curvature for Riemann manifolds admitting a Riemannian submersion. We establish some inequalities for Riemannian submersions. Using by these inequalities, we get some characterizations for Riemannian submersions.

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On Submanifolds in Tangent Bundle with g-natural Metric

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The class of g-natural metrics is a class of metrics on a tangent bundle that contains the Sasaki metric, the Cheeger-Gromol one, the Oproiu family and other metrics obtained from a Riemannian metric g on a given base manifold M by classical lifts.

We are going to present some results concerning submanifolds in TM that are generating either by a vector field on M or by an immersion in M.

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Equivariant Chen-Willmore Surfaces in Conformal Homogeneous 3-Spaces

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The geometry of homogeneous three spaces with rigidity four is encoded in two values: the constant curvature, c, of the base surface and the so called *bundle curvature*, r. We have recently obtained the classification of *equivariant Chen-Willmore surfaces* of this kind of homogenous spaces, by lifting *elastic curves* of the base. In particular, their qualitative behavior is again encoded in the parameters $\{c, r\}$. In this talk, we review some of the main techniques we have used to get the classification and show how the analysis of both, the *solving natural equations problem* and the *closed curve problem* for elasticae in surfaces with constant curvature, allow us to give explicit parametrizations of Chen-Willmore surfaces and Chen-Willmore tori in the above family of conformal homogeneous three structures.

New Results for Submanifolds of Generalized Complex Space Forms with a Semi-symmetric Metric Connection

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In two papers from 2010 and 2011, the authors A. Mihai and C. Ozgur obtained Chen inequalities in the case of submanifolds in real space forms, complex space forms and Sasakian space forms, with a semi-symmetric metric connection.

In this article, we find new Chen inequalities for submanifolds of a generalized complex space forms $\widetilde{M}(c, \alpha)$ (RK-manifolds of constant holomorphic sectional curvature c and of constant type α) *with a semi-symmetric metric connection*, that's means relations between the mean curvature associated with the semi-symmetric metric connection, scalar and sectional curvatures, Ricci curvatures and the constant holomorphic sectional curvature of the ambient space. The equality cases are considered.

2000 MSC: 53C40, 53C55, 53B05, 53B15.

Keywords and phrases: Generalized complex space form, semi-symmetric metric connection, Chen inequalities, Ricci curvature.

On Twisted Surfaces

Wendy GOEMANS

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This talk is about twisted surfaces. These surfaces are generated by synchronized rotations of a planar curve in its supporting plane and of this supporting plane about some axis. The resulting surfaces can be seen as generalizations of surfaces of revolution. The Möbius strip and the twisted Klein bottle are two well-known examples of twisted surfaces in Euclidean 3-space.

First, we provide an overview of what is currently known about twisted surfaces. Then, we study curvature conditions on twisted surfaces in Euclidean and Minkowski 3-space. Where possible a classification is presented, otherwise non-existence results are stated.

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Polynomial Approximation of the Solution of Boundary Value Problems for ODEs which Arise from Engineering

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Consider the system of linear differential equations

$$Y'(x) = B(x)Y(x) + C(x), \tag{1}$$

where $B(x) = (b_{i,j}(x))_{i,j=1,...,N}$, $C = (c_i(x))_{i=1,...,N}$, and $b_{i,j},\ c \in \mathcal{C}^\infty([0,1],\mathbb{R})$. Suppose that

$$D_j Y(\theta_j) = e_j,\ j = 1, ..., t. \tag{2}$$

where $D_j^T \in \mathbb{R}^N$, $e_j \in \mathbb{R}$, and $\theta_j \in [0,1]$. If the boundary value problem (1)-(2) has a unique solution $Y \in \mathcal{C}^\infty([0,1],\mathbb{R}^N)$, then, under suitable hypotheses, the solution of this problem is uniformly approximated by polynomials (see [2]-[4]). We apply this method to approximate solutions of BVP for ODEs, of this type, used in modeling engineering problems (see for example [1] or [5]).

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Flat Tensor Product Surfaces of Pseudo-Euclidean Curves

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We determine the flat tensor product surfaces of two curves in pseudo-Euclidean spaces of arbitrary dimensions. The poster is based on a joint paper to appear in Annales Polonici Matematici.

An 1–differentiable Cohomology Associated to a Vector Field

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A new cohomology, induced by a vector field, is defined on pairs of differential forms (1–differentiable forms) in a manifold. It is proved a link with the classical de Rham cohomology and a 1-differentable cohomology associated to an one form. Also, the case when the manifold is complex and the vector field is holomorphic is studied. Some basic properties of this cohomology as: Poincaré Lemma, homotopy invariance, Mayer-Vietoris sequence, relative cohomology, fiber integration for cohomology with compact supports are obtained.

2000 MSC: 14F40; 57R99; 58A10; 58A12.

Keywords and phrases: 1–differential form; cohomology; Lie derivative; vector field.

Some Characterizations on Screen Locally Conformal Coisotropic Lightlike Submanifolds of a Semi-Riemannian Manifold of Index 2

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We introduce screen Ricci curvature and screen scalar curvature of screen locally conformal coisotropic lightlike submanifolds of a semi-Riemannian manifold of index 2. We establish some general inequalities involving screen scalar curvature on these submanifolds. From these inequalities, we give some characterization on screen locally conformal coisotropic lightlike submanifolds. Furthermore, we discuss the notion of best living way on these submanifolds.

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Curves of AW(k)-type in the Equiform Geometry of the Pseudo-Galilean Space

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Frenet and Bertrand curves of AW(k)-type ($1 \leq k \leq 3$) are studied in the equiform geometry of the pseudo-Galilean space G_3^1. Furthermore, we have shown that the Bertrand curve of AW(k)-type in the equiform geometry of $G_3^1 3$ is a circular helix. Moreover, some equiform curvatures conditions of Frenet curves are given.

Submanifolds with Parallel T, ω, B and C in a Locally Decomposable Riemannian Manifold

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Recently, B. Sahin studied submanifolds in a locally product Riemannian manifold and gave a very interesting results. And we considered certain submanifolds in a locally decomposable Riemannian manifold ([4]).

In this talk, we define the parallelism of the tensor fields T, ω, B and C which are defined by (2.4) and (2.5) (See Definition 4.1). Then, we consider submanifolds with parallel T, ω, B and C in a locally decomposable Riemannian manifold.

2000 MSC: 53C40.

Keywords and phrases: Locally decomposable Riemannian manifold, F-invariant submanifold, Totally umbilic submanifold.

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On the Normal Scalar Curvature Conjecture

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The normal scalar curvature conjecture, also known as the DDVV conjecture, was stated by De Smet, Dillen, Verstraelen and Vrancken in [Arch. Math. (Brno) **35** (1999), 115–128]. It was proven by Lu [J. Funct. Anal. **261** (2011), 1284–1308] and by Ge and Tang [Pacific J. Math. **237** (2008), 87–95], independently.

Recently, in [Nonlinear Anal. **95** (2014), 714–720], we proved the DDVV inequality, also known as generalized Wintgen inequality, for Lagrangian submanifolds in complex space forms. Some applications are given. For instance, we derive that a Lagrangian submanifold satisfying the equality case identically is a Chen submanifold. Also we state such an inequality for slant submanifolds in complex space forms.

On Product Complex Cartan Spaces

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In this paper we consider the product of two complex Cartan manifolds, the outcome being a class of product complex Cartan spaces. Then, we study the relationships between the geometric objects of a product complex Cartan space and its components, (e.g. Chern-Cartan complex nonlinear connection, Cartan tensors). By means of these, we establish the necessary and sufficient conditions under which a product complex Cartan space is Landsberg-Cartan or Berwald-Cartan or it has other some properties.

Riemannian Submersions Induced by Affine Actions on Lie Groups

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A free, isometric action on a Riemannian manifold induces a (homogeneous) Riemannian submersion whose leaves are given by the orbits of the action. Due to their curvature non-decreasing nature, Riemannian submersions have been used extensively to construct new metrics of non-negative and positive curvature out of old ones. We will show that if we consider affine instead of isometric actions on Lie groups with left invariant metrics, then, under certain conditions, the induced submersions remain Riemannian. In particular, this allows for the construction of new examples of non-homogeneous Riemannian submersions.

Spin^c Structures and Isometric Immersions of Riemannian Manifolds

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We establish a correspondence between the existence of a Spin^c Killing spinor on homogeneous 3-dimensional manifolds $\mathbb{E}^*(\kappa,\tau)$ with 4-dimensional isometry group and isometric immersions of $\mathbb{E}^*(\kappa,\tau)$ into the complex space form $\mathbb{M}^4(c)$ of constant holomorphic sectional curvature $4c$, for some $c \in \mathbb{R}^*$. As applications, we show the non-existence of totally umbilic surfaces in $\mathbb{E}^*(\kappa,\tau)$ and we give necessary and sufficient geometric conditions to immerse a 3-dimensional Sasaki manifold into $\mathbb{M}^4(c)$.

The Piecewise Hopf Link Undulator Model

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In this paper we introduce a Hopf Link undulator structure. Analytical expressions for the magnetic field of this type of insertion devices for the free electron laser is given. This type of knot structures assure the electron trajectories winding for a new proposed free electron laser.

On Absolutely Umbilical Hypersurfaces in the Euclidean Ambient Space

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In the last two decades, there have been important advances in the study of new curvature invariants. A natural question is to investigate which fundamental algebraic inequalities yield meaningful consequences in terms of curvature. Recently, a new curvature invariant was studied: the amalgamatic mean curvature. We will discuss its geometric meaning and will show how a new class of geometric objects is obtained: the absolutely umbilical hypersurfaces. Then we will survey a series of new results obtained in the investigation of natural extensions of absolutely umbilical hypersurfaces.

Doubly Warped Product Submanifolds in Generalized Complex Space Forms

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In this paper, we establish an inequality between the warping functions f_1 and f_2 (intrinsic structures) and the squared mean curvature $||H||^2$ and the holomorphic sectional curvature c (extrinsic structures) for doubly warped product submanifolds $M =_{f_2} M_1 \times_{f_1} M_2$ in any generalized complex space form $\widetilde{M}(c, \alpha)$. Some applications are derived.

On Some Classes of Slant Curves in Trans-Sasakian Manifolds

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We consider slant curves in trans-Sasakian manifolds with C-parallel and C-proper mean curvature vector field in the tangent and normal bundles. We find the characterizations of the curvatures of these types curves and give some examples.

Aplications of Nonstandard Analysis in Mathematical Economics

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The present talk is centered on using of some tools coming from Nonstandard Analysis in order to test if the analogues for Large Finite Economies (modeled as a special kind of Mathematical Games with a la-rge finite number of players) of some Theorems from the Continuous Economies (modeled as Mathematical Games with a continuum of players) remains true. The Continuous Economies were introduced

by the Nobel laureate in Economy, Robert Aumann in order to allow us to use the mathematical methods of Topology, Differential and Integral, or Variational Calculus, avoiding in such a way the problems aris-ing from the complexity of the calculus (very difficult in many cases) when dealing with large, finite, eco- nomies, like the economies of USA or European Union. If the finite analogue remains true (for a large number of traders), those theorems (from Continuum Economics) can be used when taking economical decisions, otherwise not. We develop a testing method, exemplifying in both cases (an example of a Theorem true for continuous economies, but false for any finite economy already belongs to R. Aumann). Similar methods can be applied in other arias, like Continuum Mechanics, but we do not insist in this talk.

The Poincaré-Cartan Form and Conservative Numerical Methods for PDE

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We study the Poincaré-Cartan form on k jet spaces and its implication on construction of conservative numerical methods for PDE.

Sedimentation and Morphology 3D Mathematical Modelling for Pitesti Reservoir

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Romania is known for the large quantities of alluvia that rivers are carrying. On average 50 tons per year and kilometer of solid flow, that turns, related to the area of the country, in about 200 t/ha and year. Approximately 50% of the total annual volume of silt drains in less than one month, according to the flood period.

In this paper the sedimentation and morphology modeling for Pitesti reservoir case study are reviewed. The novelty point is the computational 3D modeling, which is seldom used in similar analysis because of the involved complexity, consists in the morphology of the mathematical model with each time frame.

The mathematical modeling of the silting and alluvia movement phenomena is based on the meshing of the analyzed domain, using finite difference method. The case study has its focus on Pitesti reservoir, in Arges river basin, with an estimate of over 97% of the total volume filled with silt.

Keywords: Morphological Processes, River Sedimentation, Mathematical Modelling.

Non Commutative Inequalities in Operator Algebras

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Standard inequalities in $C^\star$-algebras are defined with respect to positive elements and positive maps.We emphasize the fact that basically, inequalities are strongly connected to commutativity. We give some examples to show the influence of commutativity onto inequalities in $C^\star$-algebras. Since completely positive maps are the adecquate tool in non-commutative cases, it seems natural to consider a similar notion for inner products on $C^\star$-algebras, regarded as Hilbert $C^\star$-modules. This leads to the left and right type of n-positive inner products, $\mathcal{L}_n, \mathcal{R}_n$, and completely positive $\mathcal{L} = \cap\mathcal{L}_n$, $\mathcal{R} = \cap\mathcal{R}_n$. The particular case $\mathcal{L}_2, \mathcal{R}_2$ produces a non-commutative version of the well-known Schwarz inequality. The standard inner products $< a, b >_L = b^\star a$ and $< a, b >_R = ab^\star$ are the basic examples for the left and right types. Therefore it is important to determine how many other such completely positive inner products can be defined on a $C^\star$-algebras. Their class describes the non-commutativity of the algebra.

String-like Photon-Electron Interaction

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It is well known that the electron is an elementary particle: this means that it can not spin up, like proton for example. But, on the other side, neither photons nor electric particles are permanent entities, they may come and go in various numbers. In fact all charged particles are dressed in a cloud of so-called virtual photons, rather like ephemeral bees around a subatomic hive. We try to describe the interaction between a photon and a free electron via the string-like theory.

On 3-dimensional Contact Metric Generalized (k, μ)-space-forms

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Geometric Inequalities for Submanifolds in Sasakian Space Forms

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Legendrian and special contact slant submanifolds in Sasakian space forms play an important role in contact geometry.

This article has two parts.

In the first part of it, we obtain Chen like inequalities for Legendrian submanifolds in Sasakian space forms, i.e relationships between intrinsic and intrisic invariants of such submanifolds, involving the scalar curvature and Chen first invariant, respectively, and the squared mean curvature and the $\phi-$sectional curvature of the ambient space.

In the last part, we obtain Chen like inequalities for a special contact slant submanifold M in a Sasakian space form $\widetilde{M}(c)$, in terms of the main extrinsic invariant, namely the squared mean curvature.

Scalar and Ricci Curvatures of Special Contact Slant Submanifolds in Sasakian Space Forms

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We prove a Chen inequality involving the scalar curvature and a Chen-Ricci inequality for special contact slant submanifolds of Sasakian space forms, as the contact versions of the inequalities obtained by the first author in [1] and by both authors in [2], respectively. The poster is based a the joint paper published in Advances in Geometry **14** (2014), 147-159.

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Parametric Analysis Using Stochastic Approaches in Abaqus

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Worldwide, landslides pose a major treat to social and economic activities, so that sustained efforts have been made in locating and predicting the occurrence of this kind of unwanted events, such as land-based or even space borne remote sensing. The triggering mechanisms are often considered to be merely the sum of unforeseen events that lead to the landslide, but it may not always be the case. As a consequence, stability assessment trough stochastic approaches appears to be the most dependable method, even though the reliability analysis models adopted will have to be different than the ones usually found in specific literature. In turn, the results of a stochastic analysis shift their meaning from the safety factor or stability margin to probability of failure. Such an analysis was initially used in Limit Equilibrium Method computation, yet the intrinsic disadvantages of the manner the stability was equated lead to its demise. The aim of the article is to offer an insight to stochastic parametric analysis done in ABAQUS. Using randomised parameters, the new Finite Element Method formulation, gives realistic results in a wide range of soil instability types. The scripting features of the mentioned FEM software will also be used in order to reduce the redundant character of such an analysis.

Keywords: parametric analysis, stochastic analysis, reliability models, FEM.

The Role of Reputation in the Struggle between Online versus Offline Commerce

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When a consumer wants to buy something it is very important for him to trust the company (product and deliver time) he chose. Lack of trust and moral hazard are two serious impediments to efficiency and growth. Corporate reputation and competition are two important things that can fix this problem. It is easily to show

that competition relies on corporate reputation. In this paper I will speak about the influence of corporate reputation on commerce and I will show the role of reputation in the struggle between two companies.

Numerical Analysis of the Seepage Phenomena at Gura Apelor Dam Using a 3D Finite Element Model

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Gura Apelor Dam, completed in 1986 is a 168 m high rock-fill dam with central impervious core. The dam provides a reservoir with a total volume of 230 million cubic meters for Gura Rului Hydropower Development. Foundation treatment consisted of a centreline grout curtain. Analyses of seepage through the dam foundation and abutments were required to estimate seepage losses and hydraulic gradients at critical location and the constructive measures to limit these phenomena. The flow regime in foundation was studied by numerical analyses using the finite element method. A 3 D finite element model for dam foundation system was built so as to allow predictions of seepage flow and maximum hydraulic gradients and can be easily adapted to different scenarios corresponding to the operational levels in the reservoir, based on monitoring data and in certain conditions of implementation for the grout curtain. The sensitivity analyses provide the background for deciding the extent of foundation treatment required to satisfy design and safety criteria and its efficiency.

Keywords: 3D mathematical modelling, finite elements, grout curtain, finite element.

Sand Liquefaction Potential Assessment by Normalized Skew Surface State Boundary

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Liquefaction is a phenomenon occurring in saturated cohesionless soils consisting of the pore water pressure increase due to mechanical actions so that the mineral grains become afloat. This state is characterised by the cancellation of the shearing strength (the behaviour being rather of a non-Newtonian viscous liquid) and the decrease of the linear deformation modulus. However, the phenomenon depends both on a set of physical parameters such as grain size distribution, porosity, shape factor, roughness and mechanical parameters, the most important being confining stress, load amplitude and frequency. In order to assess the liquefaction potential of a sand, a new multi criterion surface state boundary is proposed describing the variation of the

linear deformation modulus with the three aforementioned mechanical parameters. The tests were carried-out in a cyclic triaxial compression equipment on two types of sand prepared in the loosest state by pluviation under water. These results become the starting point of numerical analyses taking into account the discrete nature of the material as cause for liquefaction.

Keywords: Liquefaction, normalized state boundary, cyclic triaxial test, low frequency hysteresis.

On the Generalization of the Skew-symmetric Einstein Model by Using Conjugate Connections. Associated Invariants

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We define g-equivalent and consistent Einstein models. The classical Einstein model is a particular case. The skew-symmetric Einstein model is generalized, but remains consistent. Associated invariants are obtained.

This talk continues the work "Cosmology, the General Theory of Relativity and G-conjugated Models" (27-th Texas Symposium, Dallas, December 8-13, 2013).

Discrete Curvatures of Triangular Meshes that Approximate Smooth Surfaces: the Case of Terrain Data

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In recent years, discrete differential geometry operators emerged as a powerful tool in computer graphics, robotics, pattern recognition and other related scientific fields. Transferring well-established concepts such as Gaussian and mean curvature from the smooth to the discrete case is not straightforward and several alternative approaches were proposed in the literature. As the researches point out, the computation of the curvatures for polygonal meshes approximating smooth surfaces can be complemented by accuracy estimates. Thus, comparisons between the various methods were already realized in the case of range data collected from 'synthetic' surfaces such as plane, sphere, cone and cylinder. The aim of this study is to perform similar computations and comparisons for true terrain data, acquired through *in situ* measurements, for which the underlying geometry is unknown. Starting from digital elevation models, a discrete height function can be generated. The latter yields a smooth surface, obtained by applying standard interpolation spline techniques. Further, points randomly sampled on the surface give rise to a triangular irregular network. For this triangular mesh, the values of the discrete Gaussian and mean curvatures, as provided by standard discrete methods (paraboloid fitting, Gauss-Bonnet scheme, tensor-based approach), are computed, compared with the 'true' smooth curvatures and interpreted in relationship with the terrain features.

Geometry of Submanifolds

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Flat Almost Complex Surfaces in the Nearly Kähler $S^3 \times S^3$

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Nearly Kähler manifolds have been studies intensively in the 1970's by Gray. These nearly Kähler manifolds are almost Hermitian manifolds for which the tensor field ∇J is skew-symmetric. In particular, the almost complex structure is non-integrable if the manifold is non-Kähler. A well known example is the nearly Kähler 6-dimensional sphere, whose complex structure J can be defined in terms of the vector cross product on $\mathbb{R}^7$. Recently it has been shown by Butruille that the only homogeneous 6-dimensional nearly Kähler manifolds are the nearly Kähler 6-sphere, the nearly Kähler $S^3 \times S^3$, the projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. All these spaces are compact 3-symmetric spaces

There are two natural types of submanifolds of nearly Kähler (or more generally, almost Hermitian) manifolds, namely almost complex and totally real submanifolds. Almost complex submanifolds are submanifolds whose tangent spaces are invariant under J. Almost complex submanifolds in the nearly Kähler manifold S^6 have been studied by many authors. Also in the nearly Kähler $\mathbb{C}P^3$ some results have been obtained by Xu Feng.

In this talk we show how to study almost complex submanifolds of $S^3 \times S^3$. Compact 6-dimensional non-Kähler nearly Kähler manifolds do not admit 4-dimensional almost complex submanifolds, so the almost complex submanifolds are surfaces.

Previously, a classification of the totally geodesic almost complex surfaces has been ontained as well as the result that an almost complex immersion of a topological 2 sphere is necessary totally geodesic. In this lecture we will show how to obtain a classification, as well as explicit expressions of all almost complex flat surfaces.

Recent Progress in Riemannian Geometry, Complex Geometry and Topology

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We will discuss some recent progress in Riemannian geometry, complex geometry and topology. Their interconnectedness and examples will be provided.

On Ideal Submanifolds of Real Space Forms with Type Number ≤ 2

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(Based on a joint work with Bang-Yen CHEN)

It is known that an ideal immersion of a Riemannian manifold into a real space form is an isometric immersion which produces the least possible amount of tension from the ambient space at each point of the submanifold and the classification problem of ideal submanifolds in real space forms is a very challenging problem. In this talk, taking into account this problem, we give the classification of all non-minimal ideal submanifolds of real space forms with type number ≤ 2.

The Geodesics and Harmonic Maps between Two Second Order Holomorphic Jets Bundle

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In the geometry of the holomorphic jets bundle of order two $J^{(2,0)}M$, we have studied a special $N-$ linear connection, named the Chern-Lagrange connection.

We study the first variation of the energy for the curves from J(2,0)M, depending on the general metric structure G and a fixed N linear connection D, and we obtain the geodesic curves of the energy.

We consider a holomorphic function $f : M \to N$ between two complex manifolds, which carries the curves from $J^{(2,0)}M$ into curves on $J^{(2,0)}N$, and we find when this mapping is harmonic.